

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 6

LEFEL 6

EXAMINATION PERIOD
JANUARY 2017

CYFNOD ARHOLIADAU
LONAWR 2017

MODULE CODE: M3X7200
ADVANCED STRESS ANALYSIS (PART TIME) (RESIT)

TIME: 2 HOURS

ANSWER THREE OUT OF FOUR QUESTIONS

Equipment or materials required – None

Adnoddau neu ddefnyddiau sydd eu hangen – Dim

A formula sheet is provided for reference.

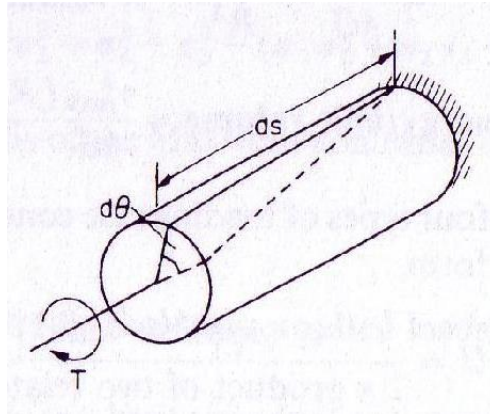
Calculators are permitted.

Assume 'g' = 9.81 m/s².

ONE

- a) (i) Consider the bar below subjected to torsion. Prove using the simple theory of elastic torsion ($\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$) that the change in angle, $d\theta$, due to the torque is given by

$d\theta = \frac{Tds}{GJ}$, where T is the Torque, ds is the length of the bar, G is the shear modulus and J is the polar second moment of area.



[2 Marks]

- (ii) Hence prove that the strain energy, U, for a bar in Torsion is given by; $U = \frac{T^2 L}{2GJ}$

[4 Marks]

- (iii) What is meant by the term Resilience? Hence from prove from the equation for a bar in tension ($U = \frac{P^2 L}{2AE}$, where P is the load applied, A is the cross sectional area and

E is the Youngs Modulus), that resilience may be defined as Resilience = $\frac{\sigma^2}{2E}$

[4 Marks]

- b) (i) A steel rod has a square cross section 20mm x 20mm and a length of 4m. Calculate the strain energy when a stress of 300MPa is produced by stretching it. (E=200GPa)

[3 Marks]

- (ii) The rod then has a metal collar fitted at the lower end, and a mass, m, is dropped onto the collar. Show, by considering the energy transferred from the falling weight to the rod, that the height, h, is related to the stress, σ , by the following relationship;

$$h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$$

Where V is the volume of the bar, and L is the length. Assume that the extension of the bar is significant in the energy transfer.

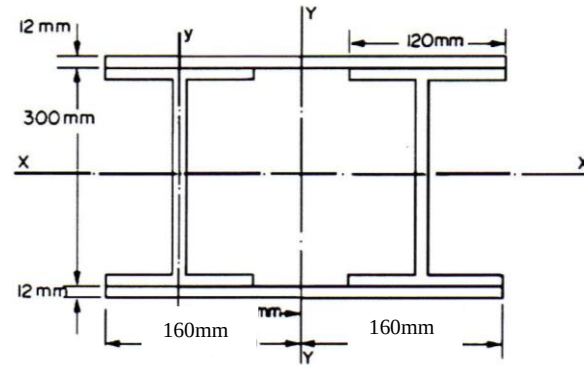
[4 Marks]

- (iii) If the stress must not exceed 75MPa, from what height could 200kg fall onto the collar, without exceeding this?

[3 Marks]

TWO

Two 300mm x 120mm I-section joists are united by 12mm thick plates as shown below to form a 7m long stanchion. Given that the Young's Modulus $E = 200 \text{ GPa}$, compressive yield stress is 320 MPa and a constant a of $1/7500$, assuming pinned ends, determine the following;



Properties of ONE joist;

$$I_{xx} = 96 \times 10^{-6} \text{ m}^4$$

$$I_{yy} = 4.2 \times 10^{-6} \text{ m}^4$$

$$A = 6 \times 10^{-3} \text{ m}^2$$

- a) Which axis will the strut buckle about?
[5 Marks]
- b) What is the radius of gyration about this axis?
[2 Marks]
- c) What is the slenderness ratio of this strut?
[2 Marks]
- d) According to Euler theory, what load will this strut buckle at?
[3 Marks]
- e) According to Rankine-Gordon theory, what load will this strut buckle at?
[3 Marks]
- f) Comment on the results of (d) and (e), and with the use of a diagram explain the difference between experimental results and buckling loads according to Euler theory.
[5 Marks]

THREE

- a. In the context on Engineering define what is meant by a fatigue failure. [2 Marks]
- b. Describe the three surface features that typify a fatigue failure. [3 Marks]
- c. With the aid of a diagram discuss the influence of increasing the mean stress on the Sn curve of a specimen. [3 Marks]
- d. Identify three theories that are used to quantify the effect of tensile mean stress on the number of cycles to failure. With the use of a diagram explain the relationship between these theories, where failures are typically seen and thus which theory is often used and the reason for this. [8 Marks]
- e. The values of the endurance limits at various stress amplitude levels for low-alloy construction steel fatigue specimens are given below:

σ_a (MPa)	N_f (cycles)
550	1,500
510	10,050
480	20,800
450	50,500
410	125,000
380	275,000

A similar specimen is subjected to the following program of cycles at the stress amplitudes stated: 3,000 at 510MPa, 12,000 at 450MPa and 80,000 at 380Pa, after which the sample remains unbroken. Using Miners law determine how many additional cycles would the specimen withstand at 480MPa prior to failure? Assume zero mean stress conditions.

[4 Marks]

FOUR

- a) A mild steel shaft of 50mm diameter is subjected to a bending moment of 1.9kNm. Using

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \text{ calculate the maximum bending stress.}$$

[3 marks]

- b) In addition to this bending stress this shaft is to carry a torque. Using $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$ show that the relationship between the shear stress, τ , and the torque, T , when carrying a torque alone, is given by $\tau=40700T$

[3 marks]

- c) If the yield point of the steel in a simple tension test is 200MPa, find the maximum torque that can be applied according to,

- i) The maximum shear stress theory of yielding

[7 marks]

- ii) The Von-Mises theory of yielding

[7 marks]

FORMULA SHEET

Energy

$$U = \frac{P^2 L}{2AE}$$

Strain energy in tension

$$U = \frac{Q^2 L}{2AG}$$

Strain energy in shear

$$U = \frac{M^2 L}{2EI}$$

Strain energy in bending

$$U = \frac{T^2 L}{2GJ}$$

Strain energy in torsion

$$\text{P.E.} = mgh$$

Potential energy

$$\text{K.E.} = \frac{1}{2} mv^2$$

Kinetic energy

Bending/Torsion

$$I_{NA} = \frac{BD^3}{12}$$

Second moment of area of a rectangle

$$I_{NA} = I_G + Ah^2$$

Parallel axis theorem

$$I = \frac{\pi d^4}{64}$$

The second moment of area of a circular shaft.

$$J = \frac{\pi D^4}{32}$$

The polar second moment of area of a circular shaft

Buckling

$$\frac{L}{k}$$

Slenderness Ratio

$$k = \sqrt{\frac{I}{A}}$$

Radius of gyration

$$P_e = \frac{\pi^2 EI}{L^2}$$

Buckling: pinned-pinned

$$P_e = \frac{\pi^2 EI}{4L^2}$$

Buckling: fixed-free

$$P_e = \frac{4\pi^2 EI}{L^2}$$

Buckling: fixed-fixed

$$P_e = \frac{2\pi^2 EI}{L^2}$$

Buckling: fixed-pinned

$$P_R = \frac{\sigma_y A}{1 + a(L/k)^2}$$

Rankine Load

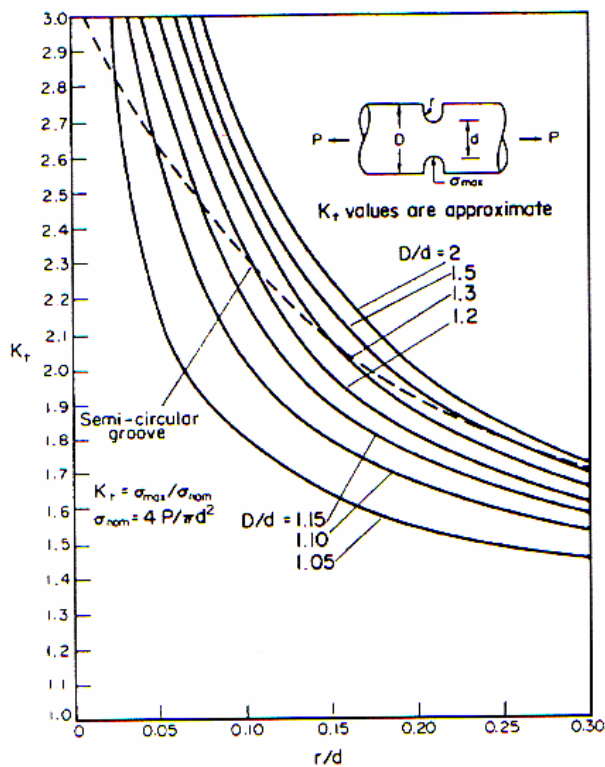
Theories of yielding

Theory	Value in tension test at failure	Value in complex stress system	Criterion for failure
Maximum principal stress (<i>Rankine</i>)	σ_y	σ_1	$\sigma_1 = \sigma_y$
Maximum shear stress (<i>Guest-Tresca</i>)	$\frac{1}{2}\sigma_y$	$\frac{1}{2}(\sigma_1 - \sigma_3)$	$\sigma_1 - \sigma_3 = \sigma_y$
Maximum principal strain (<i>Saint-Venant</i>)	$\frac{\sigma_y}{E}$	$\frac{\sigma_1 - \nu \sigma_2 - \nu \sigma_3}{E}$	$\sigma_1 - \nu \sigma_2 - \nu \sigma_3 = \sigma_y$
Total strain energy per unit volume (<i>Haigh</i>)	$\frac{\sigma_y^2}{2E}$	$\frac{1}{2}[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1))]$	$\frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}{2} = \frac{\sigma_y^2}{2}$
Shear strain energy per unit volume Distortion energy theory (<i>Maxwell-Huber-von Mises</i>)	$\frac{\sigma_y^2}{6G}$	$\frac{1}{12G}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$	$\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \sigma_y^2$
Modified shear Stress/Internal friction theory (<i>Mohr</i>)			$\frac{\sigma_2 + \sigma_3}{\sigma_{yt} \sigma_{yc}} = 1$

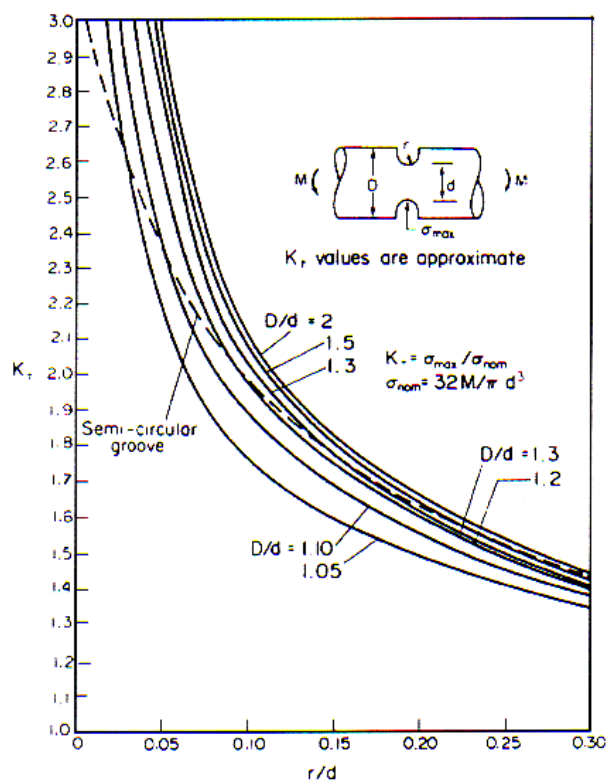
Complex stress

$$\sigma_{1,2} = \frac{1}{2} \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$

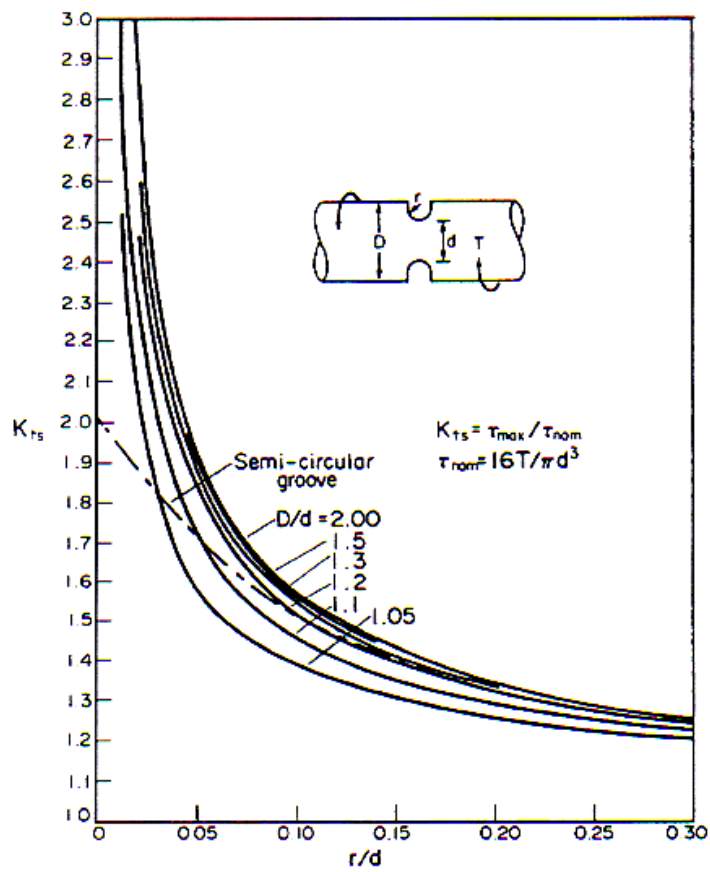
Stress Concentrations



K_t for a round bar with a U groove, subject to TENSION



K_t for a round bar with a U groove, subject to BENDING



K_t for a round bar
with a U groove,
subject to
TORSION

Diwedd/End